Mathematical Reasoning

STATEMENT Section - 1

We communicate our ideas or thoughts with the help of sentences in particular languages.

Note : A true statement is known as a valid statement.

A false statement is known as an invalid statement.

A statement cannot be both true as well as false at the same time.

Mathematical identities are considered to be statements.

(a) Type of Sentences :

Following types of sentences are generally used in our daily normal life.

(i) Assertive Sentence : A sentence that makes an assertion is called as assertive sentence or a declarative sentence.

Example : 1. The sun is a star. **2.** Jaipur is the capital of Rajasthan.

(ii) Optative Sentence : A sentence that expresses a wish is called an interrogative sentence.

Example : 1. God bless you. **2.** Wish you best of luck

Truth Value of a Statement :

If a statement is true, then its truth value is true or T. If it is false then its truth value is false or F.

Such a table representing the truth values of a compound statement as dependent on its sub-statement is called a truth table

Note : For n statements, there are 2^n rows.

1. Truth table for single statement p

Number of rows = $2^1 = 2$

p
T
F

2. Truth table for two statements p and q.

Number of rows = $2^2 = 4$

p	q
T	T
T	F
F	T
F	F

Compound Statement :

A statement which is formed by combining two or more simple statements is called a compound statement.

Example : (1) This book is for mathematics and it targets AIEEE.

(2) 13 is a prime number and it is an odd number.

(d) Some properties :

1. Prove that $\sim(\sim p) = p$

Proof : Truth table :

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

i.e., first and third columns are identical

2. Prove that $\sim(p \wedge q) = \sim p \vee \sim q$

Proof : Truth table :

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

i.e., last two columns are identical

3. Prove that $\sim(p \vee q) = \sim p \wedge \sim q$

Proof : Truth table :

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

i.e., last two columns are identical.

4. Prove that $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

Proof : Truth table :

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	F	T	F	F	F	F
T	T	F	T	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	F	T	F	F	F	F
F	T	F	F	F	F	F
F	F	F	F	F	F	F

i.e., last two columns are identical

Illustration - 1 *Given the following statements*

$p : 3\times 8 = 24$; q : *Sun is a star* ; r : *Roses are yellow* ; $s : \cos 30° = \frac{\sqrt{3}}{2}$

State the truth values of the following :

(i) $p \wedge q, p \wedge r, p \wedge s, q \wedge r, q \wedge s, r \wedge s$ (ii) $p \vee q, p \vee r, p \vee s, q \vee r, q \vee s, r \vee s$

(iii) $\sim (p \vee s),\ \sim (p \vee r)$ (iv) $p \wedge r, r \vee s, q \vee s$

SOLUTION:

Here p, q, s are true statements and r is a false statement.

1. True, False, True, False, True, False
2. All are true
3. Both false
4. False, True, True

CONDITIONAL AND BICONDITIONAL STATEMENTS — Section - 3

(a) Conditional Statement :

If p and q are two statements, then statement of the form 'if p then q' is called the conditional statement and it is denoted by $p \Rightarrow q$ and is read as p implies q.

Note : $p \Rightarrow q$ is true in all cases except when p is true and q is false. $p \Rightarrow q$ also means

(i) p is sufficient for q (ii) q is necessary for p

(iii) p leads to q (iv) q is p

Truth table for a conditional statement

p	q	$p \Rightarrow q$	$q \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

(b) Contrapositive :

If p and q are true statements, then the contrapositive of the implication $p \Rightarrow q = (\sim q) \Rightarrow (\sim p)$

Proof : For any two statements p and q, let us consider the truth tables of the statements $p \Rightarrow q$ and $\sim q \Rightarrow \sim p$ as given below

Note : The statement $p \Rightarrow q$ has the truth value F only when the truth value of p has T and truth value of q has F.

Truth Table :

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(c) Some properties :

1. Prove that $p \Rightarrow q = \sim p \vee q$

Proof : truth table

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that the last two columns are identical

Illustration - 2 *Write the contrapositive of the statement. If x is non-zero, then x is four.*

SOLUTION:

Let p : x is non-zero

q : x is four

$\sim p$: x is not non-zero

$\sim q$: x is not four

$\sim q \Rightarrow \sim p$: If x is not four, then x is not non-zero

Converse : If p and q are two statements then the converse of $p \Rightarrow q$ is $q \Rightarrow p$. But $p \Rightarrow q \neq q \Rightarrow p$

(d) Biconditional Statements :

If p and q are two statements, then statement of the form 'p if and only if q' is called the biconditional statement and it is denoted by $p \Leftrightarrow q$

It means :

(i) p is a necessary and sufficient condition for q

(ii) q is a necessary and sufficient condition of p

(iii) If p then q and if q then p

(iv) q if and only if p.

Note : **1.** $p \Leftrightarrow q$ is true if both p and q have the same truth value *i.e.*, if either both p and q are true or both are true.

2. $p \Leftrightarrow q$ is false if p and q have opposite truth values.

Truth table for a biconditional statement:

p	q	$p \Leftrightarrow q$
T	T	T
F	T	F
T	F	F
F	F	T

(e) Some properties :

1. Prove that $(p \Leftrightarrow q) \Leftrightarrow r = p \Leftrightarrow (q \Leftrightarrow r)$

Proof : Truth table :

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	$(p \Leftrightarrow q) \Leftrightarrow r$	$p \Leftrightarrow (q \Leftrightarrow r)$
T	T	T	T	T	T	T
T	F	T	F	F	F	F
T	T	F	T	F	F	F
T	F	F	F	T	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	T
F	T	F	F	F	T	T
F	F	F	T	T	F	F

We observe that the last two columns are identical.

Illustration - 3 *The following statements are biconditional statements.*

SOLUTION :

(a) A triangle is isosceles if and only if two of its sides are equal.

(b) A number is divisible by 3 if and only if the sum of the digits forming the number is divisible by 3.

TAUTOLOGIES AND FALLACIES (CONTRADICTIONS) — Section - 4

(a) Tautologies :

This is a statement which is always true of all truth values of its components.

Illustration - 4 *Consider* $p \vee \sim p$

SOLUTION: **Truth table :**

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

We observe that last column is always true

(b) Fallacy (Contradiction) :

This is a statement which is always false for all truth values of its components.

Illustration - 5 *Consider* $p \wedge \sim p$

SOLUTION: **Truth table :**

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

We observe that last column is always false

Valid Argument :

An Argument is said to be a valid argument if the conclusion p is true whenever all the hypothesis $p_1, p_2, ..., p_n$ are true or equivalently argument is valid when it is a tautology, otherwise the argument is called an invalid argument.

Method of testing the validity of arguments :

(1) Construct the truth table for conditional statements $p_1 \wedge p_2 \wedge p_3 \wedge ... \wedge p_n \to p$

(2) Check the last column of truth table. If the last column contains T only then the given arguments is valid. Otherwise, it is an invalid argument.

Illustration - 6 *Consider* $p \vee \sim (p \wedge q)$

SOLUTION:

Truth table :

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Since the last column shows T's only. therefore $p \vee \sim (p \wedge q)$ is a tautology.

Illustration - 7 $(p \wedge \sim q) \wedge (\sim p \wedge q)$ *is :*

(A) *tautology* **(B)** *a contradiction*

(C) *tautology and contradiction* **(D)** *neither a tautology nor a contradiction*

SOLUTION : (B)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

In the last column of the truth table, all entries are F.

$\Rightarrow$ The given statement is a contradiction.

Illustration - 8 *If p and q are two statements, then* $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ *is a*

(A) *contradiction* **(B)** *tautology* **(C)** *Neither* **(A)** *nor* **(B)** **(D)** *None of the above*

SOLUTION : (B)

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Hence, given proposition is a tautology.

Duality :

Two compound statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$. The connectives $\wedge$ and $\vee$ are also called duals of each other.

Duality Symbol :

Let $S(p, q) = p \wedge q$ be a compound statement. Then $S^*(p, q) = p \vee q$, where $S^*(p, q)$ is the dual statement of $S(p, q)$

★

Illustration - 9 *Write the dual of the compound statement.*

Ram and Shyam cannot read English.

SOLUTION:

Let p : Ram cannot read English

q : Shyam cannot read English

$\therefore$ Given compound statement is $p \wedge q$

Its dual $= p \vee q$ *i.e.*, One of Ram or Shyam cannot read English.

Algebra of Statements :

Statements satisfy many laws some of which are given below :

(a) Idempotent Laws : If p is any statement then :

(i) $p \vee p \equiv p$ **(ii)** $p \wedge p \equiv p$

(b) Associative Laws : If p, q, r are any three statements, then :

(i) $p \vee (q \vee r) = (p \vee q) \vee r$ **(ii)** $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

(c) Commutative Laws : If p, q are any two statements, then :

(i) $p \vee q = q \vee p$ **(ii)** $p \wedge q = q \wedge p$

(d) Distributive Laws : If p, q, r are three statements, then :

(i) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ **(ii)** $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(e) Identity Laws : If p is any statement, t is tautology and c is a contradiction, then :

(i) $p \vee t = t$ **(ii)** $p \wedge t = p$

(iii) $p \vee c = p$ **(iv)** $p \wedge c = c$

(f) Complement Laws : If t is a tautology, c is a contradiction and p is any statement, then :

(i) $p \vee (\sim p) = t$ **(ii)** $p \wedge (\sim p) = c$

(iii) $\sim t = c$ **(iv)** $\sim c = t$

(g) Involution Law : If p is any statement, then $\sim(\sim p) = p$

★ **(h)** De Morgan's Law : If p and q are two statements, then :

(i) $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$ **(ii)** $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

Use of Venn Diagrams in Logic :

An argument is the assertion that statement S follows from other statements $S_1, S_2, \ldots$ etc.

The statement S is called the conclusion and the statements $S_1, S_2, \ldots. S_n$ and conclusion S is said to be valid if S is true whenever all $S_1, S_2, S_3, \ldots S_n$ are true.

★

Illustration - 10 *Represent the truth of the statement, 'All rational numbers are real numbers' by means of a Venn diagram.*

SOLUTION:

Let Q denotes the set of all rational numbers and R denote the set of all real numbers. Let U denotes the Universal set.

$(\because Q \subseteq R)$

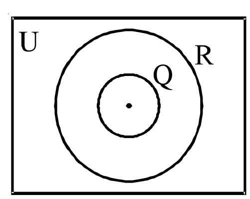


Illustration - 11 *Which of the following are statements?*

(*i*)	*The earth is round*	(*ii*)	$7 + 5 < 9$	(*iii*)	*Is 5 a positive integer?*	(*iv*)	*Come here, Rahul!*
(*v*)	*Study logic*	(*vi*)	$6 + x = 0$	(*vii*)	*It is raining here*	(*viii*)	*Sonu is a kind boy.*

SOLUTION:

(*i*) It is a declarative sentence, which is clearly true.

Therefore, it is a true statement.

(*ii*) It is a declarative sentence, which is clearly false.

Therefore, it is a false statement.

(*iii*) It is an interrogative sentence and therefore, it is not a statement.

(*iv*) It is an imperative sentence and therefore, it is not a statement.

(*v*) it is an imperative sentence and therefore, it is not a statement.

(*vi*) Without knowing the value of x, we do not know whether the given sentence is true or false. So, it is a sentence but not a statement.

(*vii*) The word 'here' is ambiguous. So it is not a statement.

(*viii*) He may be kind to one but not to others. So, it is not a statement.

Illustration - 12 *Which of the following are statements ?*

(*i*)	*Sheela is a beautiful girl*	(*ii*)	*Rohit is inside his house*
(*iii*)	*Reenu is doing her homework*	(*iv*)	*There will be snowfall in Shimla in December.*

SOLUTION:

Each one of the sentences (*i*), (*ii*) and (*iii*) is a declarative sentence which is either true or false but not both.
So, each one of these sentences is a statement.
(*iv*) is a statement, since it is a declarative sentence which is either true or false but both, although we would have to wait until December to find out it is true or false.

Illustration - 13 *Which of the following sentences are statements?*

(*i*)	*How black is the dog!*	(*ii*)	*Is Pummy, a black dog ?*
(*iii*)	*Bring Pummy, the black dog, here.*	(*iv*)	*The dog Pummy is black in colour.*

SOLUTION:

(*i*) Being an exclamatory sentence, it is not a statement.
(*ii*) Being an interrogative sentence, it is not a statement.
(*iii*) Being an imperative sentence, it is not a statement.
(*iv*) Being a declarative sentence which is either true or false but not both, it is a statement.

Illustration - 14 *Write the negation of :*

(i) *For all positive integers n,* $n^2 + 41n + 41$ *is a prime number*

(ii) *There is some integer k for which* $2k = 6$

(iii) *All the students of this school live in the hostel*

(iv) *There is a real number which is not a complex number*

(v) *None of the students of this class has passed*

(vi) *Every student has paid the fees.*

SOLUTION:

The required negation is :

(i) There is at least one positive integer n for which $n^2 + 41n + 41$ is not prime.

(ii) For all integers k, $2k \neq 6$.

(iii) At least one student of this school does not live in the hostel.

(iv) Every real number is a complex number.

(v) At least one student of this class has passed.

(vi) At least one student has not paid the fees.

Illustration - 15 *Write down the truth value of each of the following statements :*

(i) $4 + 3 = 7$ & $6 > 7$ *(ii)* $5 + 4 > 9$ & $5 < 9$ *(iii)* $3 + 3 = 6$ & $3 \geq 3$ *(iv)* $3 > 5$ & $1 > 2$

SOLUTION:

(i) Let $p : 4 + 3 = 7$

$q : 6 > 7$.

Then, $p \wedge q : 4 + 3 = 7$ and $6 > 7$.

Here, p is true and q is false, and therefore, $p \wedge q$ is false. Hence, the given statement is false and so its truth value is F.

(ii) Let $p : 5 + 4 > 9$

$q : 5 < 9$.

Then, $p \wedge q : 5 + 4 > 9$ and $5 < 9$.

Here, p is false and q is true, and therefore, $p \wedge q$ is false.

Hence, the given statement is false and so its truth value is F.

(iii) Let $p : 3 + 3 = 6$

$q : 3 \,^{3}\, 3$.

Then, $p \wedge q : 3 + 3 = 6$ and $3 \geq 3$.

Here, p is true and q is true, and therefore, $p \wedge q$ is true.

Hence, the given statement is true and so its truth value is T.

(iv) Let $p : 3 > 5$

$q : 1 > 2$.

Then, $p \wedge q : 3 > 5$ and $1 > 2$.

Here, p is false and q is false, and therefore, $p \wedge q$ is false.

Hence, the given statement is false and so its truth value is F.

Illustration - 16 *Write the truth value of each of the following.*

(i) 5 < 7 or 8 > 10.

(ii) 4 + 5 = 8 or 4 + 5 = 9.

(iii) (1 + *i*) *is a real number or it is a complex number.*

(iv) *Every quadratic equation has one real root or two real roots.*

SOLUTION:

(i) Let $p : 5 < 7$

$q : 8 > 10.$

Then, $p \vee q : 5 < 7$ or $8 > 10$.

Here p is true and q is false, and therefore, $p \vee q$ is true.

Hence, the given statement is true, and its truth values is T.

(ii) Let $p : 4 + 5 = 8$

$q : 4 + 5 = 9.$

Then, $p \vee q : 4 + 5 = 8$ or $4 + 5 = 9$.

Here, p is false and q is true, and therefore, $p \vee q$ is true.

Hence, the given statement is true, and its truth value is T.

(iii) Let $p : (1 + i)$ is a real number

$q : (1 + i)$ is a complex number.

Then, $p \vee q : (1 + i)$ is a real number or it is a complex number.

Here, p is false and q is true, and therefore, $p \vee q$ is true.

Hence, the given statement is true and so its truth value is T.

(iv) Let p : Every quadratic equation has one real root

q : Every quadratic equation has two real roots.

$\therefore$ $p \vee q$: Every quadratic equation has one real root or two real roots.

Here, p is false and q is false and therefore, $p \vee q$ is false.

Hence, the given statement is false and do its truth value is F.

Illustration - 17 *Write down the truth value of each of the following implications* :

(i) *If* $3 + 2 = 7$ *then Paris is the capital of India.*

(ii) *If* $3 + 4 = 7$ *then* $3 > 7$.

(iii) *If* $4 > 5$ *then* $5 < 6$.

(iv) *If* $7 > 3$ *then* $6 < 14$.

SOLUTION:

(i) Let $p : 3 + 2 = 7$

q : Paris is the capital of India.

$\therefore$ $p \Rightarrow q$: If $3 + 2 = 7$ then Paris is the capital of India.

Here, p is false and q is false, and therefore, $p \Rightarrow q$ is true.

Hence, the given statement is true, and its truth value is T.

(ii) Let $p: 3+4=7$

$q: 3>7.$

$\therefore$ $p \Rightarrow q$: If $3+4=7$ then $3>7$.

Here, p is true and q is false, and therefore, $p \Rightarrow q$ is false.

Hence, the given statement is false, and its truth value is F.

(iii) Let $p: 4>5$

$q: 5<6.$

$\therefore$ $p \Rightarrow q$: If $4>5$ then $5<6$.

Here, p is false and q is true, and therefore, $p \Rightarrow q$ is true.

Hence, the given statement is true, and its truth value is T.

(iv) Let $p: 7>3$

$q: 6<14.$

$\therefore$ $p \Rightarrow q$: If $7>3$ then $6<14$.

Here, p is true and q is true, and therefore, $p \Rightarrow q$ is true.

Hence, the given statement is true, and its truth value is T.

Illustration - 18 *Write the truth value of each of the following bi-conditional statements :*

(i) $4 > 2$ *if only if* $0 < (4 - 2)$ ***(ii)*** $3 < 2$ *if and only if* $2 < 1$

SOLUTION:

(i) Let $p: 4>2$ and $q: 0<(4-2)$.

The given statement is $p \Leftrightarrow q$.

Here both p and q are true and therefore, $p \Leftrightarrow q$ is true.

Hence, the given statement is true, and its value is T.

(ii) Let $p: 3<2$ and $q: 2<1$.

Then, the given statement is $p \Leftrightarrow q$.

Here both p and q are false and therefore, $p \Leftrightarrow q$ is true.

Hence, the truth value of the given statement is T.

Illustration - 19 *Write the negation of :*

(i) *He swims if and only if the water is warm.*

(ii) *India will be prosperous if and only if its citizens are industrious.*

(iii) *A triangle is equilateral if and only if it is equiangular.*

(iv) *Sets A and B are equal if and only if* $(A \subseteq B$ *and* $B \subseteq A)$.

(v) $|a| < 2$ *if and only if* $(a > -2$ *and* $a < 2)$.

SOLUTION:

(i) Let p : He swims and q : The water is warm.

The given statement is $(p \Leftrightarrow q)$.

So, its negation is $(p \wedge \sim q) \vee (q \wedge \sim p)$, given by :

Either he swims and the water is not warm, or the water is warm and he does not swim.

(ii) Let p : India will be prosperous and q : The citizens of India are industrious.

The given statement is $(p \Leftrightarrow q)$.

Its negation is $(p \wedge \sim q) \vee (q \wedge \sim p)$, given by :

Either India will be prosperous and its citizens are not industrious, or the citizens of India are industrious and India will not be prosperous.

(iii) Let p : A triangle is equilateral and q : The triangle is equiangular.

Then, the given statement is $(p \Leftrightarrow q)$.

$\therefore$ the negation of the given statement is

$(p \wedge \sim q) \vee (q \wedge \sim p)$, given by :

There exists either an equilateral triangle which is not equiangular, or an equiangular triangle which is not equilateral.

(iv) Let p : Sets A and B are equal and q : $A \subseteq B$ and $B \subseteq A$.

Then, the given statement is $p \Leftrightarrow q$.

The negation of the given statement is

$(p \wedge \sim q) \vee (q \wedge \sim p)$, given by :

Either $A = B$ and $(A \not\subseteq B$ or $B \not\subseteq A)$, or $(A \subseteq B$ and $B \subseteq A)$ and $A \neq B$.

(v) Let $p : |a| < 2$ and $q : a > -2$ and $a < 2$.

The given statement is $p \Leftrightarrow q$.

$\therefore$ the negation of the given statement is $(p \wedge \sim q) \vee (q \wedge \sim p)$, given by :

Either $|a| < 2$ and $(a - 2$ or $a \geq 2)$, or $(a > -2$ and $a < 2)$ and $|a| \geq 2$.

IN-CHAPTER EXERCISE

1. Write the truth values of the following statements :

(i) $ax^2 + bx + c = 0$ may have non-real roots.

(ii) There are only finite number of integrers

(iii) The intersection of two non-empty sets is always non-empty.

(iv) The capital of America is New York.

(v) Two individuals may be relatives.

2. Write the negation of the following statements:

(i) The square of 4 is 16

(ii) 14 divides 27

(iii) Chandigarh is in Gujarat

(iv) $7 > 3$

3. Find the truth value of the statement $\sim p$ if the statement p is :

(i) For complex numbers z_1 and z_2, $|z_1 z_2| = |z_1||z_2|$

(ii) Real part of $(1+2i)^3$ is 4

(iii) $\tan(-315°) = 1$

(iv) $\sec^2 45° + \operatorname{cosec}^2 45° = 2$

4. Let p and q stand for the statements: 'Lucknow is in U.P.' and '4 divides 12' respectively. Describe the following statements:

(i) $p \wedge q$ (ii) $p \vee q$

(iii) $\sim p \wedge q$ (iv) $\sim p \vee q$

(v) $p \wedge \sim q$ (vi) $p \vee \sim q$

(vii) $\sim p \wedge \sim q$ (viii) $\sim p \vee \sim q$

5. Write the component statements of the following compound statements and check whether the compound statement is true or false.

(i) A line is straight and extends indefinitely in both directions.

(ii) 0 is less than every positive integer and every negative integer.

(iii) All living things have two legs and two eyes.

***6.** Find the truth values of the following compound statements:

(i) $(2+4=6) \wedge (2 \times 3 = 6)$

(ii) (It is false that $2+5=8$) $\wedge$ $(2 \times 5 = 20)$

(iii) (It is false that $5-2=3$) $\wedge (4 \times 3 = 12)$

(iv) $(2+5=25) \wedge$ (It is false that $5+3=8$)

***7.** Find the truth values of the following compound statements:

(i) $(p \wedge \sim q) \vee r$ (ii) $\sim p \vee (q \wedge \sim r)$

(iii) $(\sim p \wedge \sim q) \vee \sim r$ (iv) $\sim ((p \wedge q) \vee \sim r)$

8. If statements p and q are respectively: '$3 < 4$' and '$7 > 5$' then write the biconditional statements:

(i) $p \leftrightarrow q$ (ii) $\sim p \leftrightarrow q$

(iii) $p \leftrightarrow \sim q$ (iv) $\sim p \leftrightarrow \sim q$

***9.** If truth values of statements p and q are T and F respectively then write the truth values of:

(i) $p \to q$ (ii) $p \to \sim q$

(iii) $\sim p \leftrightarrow q$ (iv) $\sim (\sim p \leftrightarrow \sim q)$

***10.** Find the truth values of the following compound statements :

(i) $\sim p \leftrightarrow q$ (ii) $\sim p \leftrightarrow \sim q$

***11.** Find the truth values of the compound statement :

$(p \vee q) \leftrightarrow r$

***12.** Show that :

(i) $(p \wedge q) \to p$ is a tautology

(ii) $p \to (p \vee q)$ is a tautology

***13.** Show that :

(i) $(p \wedge q) \wedge \sim (p \wedge q)$ is a fallacy

(ii) $(p \wedge q) \wedge (\sim p \wedge \sim q)$ is a fallacy

***14.** Show that :

(i) $(p \to q) \leftrightarrow (\sim p \vee q)$ is a tautology

(ii) $\sim (p \vee q) \leftrightarrow (\sim p \wedge \sim q)$ is a tautology

***15.** Show that : $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

***16.** *Show that :* $\sim(p \leftrightarrow q) \equiv (\sim p) \leftrightarrow q \equiv p \leftrightarrow \sim q$

***17.** *Write the duals of the following compound statements :*

(i) $[(p \vee r) \vee (\sim p \vee \sim q)] \wedge r$

(ii) $[(p \vee q) \wedge \sim r] \vee (p \wedge t)$

18. *Test the validity of the following argument:*

"If today is sunday, then yesterday was saturday. Yesterday was not saturday. Therefore, today is not sunday."

19. *Test the validity of the following argument :*

" If Nidhi works hard then she will be successful. If she is successful then she will be happy. Therefore, hard work leads to happiness".

20. *Test the validity of the following argument:*

"Wages will increase if and only if there is an inflation. If there is an inflation then the cost of living will increase. Wages increased. Therefore, the cost of living will increase.

OBJECTIVE WORKSHEET

In this Objective Worksheet, you are given questions to solve. Choose the correct alternative. EACH QUESTIONS HAS ONLY ONE CORRECT OPTION.

1. Which of the following is a statement
(A) Open the door
(B) Do your homework
(C) Switch on the fan
(D) Two plus two is four

2. Which of the following is not a statement :
(A) Please do me a favour
(B) 2 is an even integer
(C) $2+1=3$
(D) The number 17 is prime

3. Negation of the conditional : "If it rains, I shall go to school" is :
(A) It rains and I shall go to school
(B) It rains and I shall not go to school
(C) It does not rains and I shall go to chool
(D) None of these

4. Negation is "$2+3=5$ and $8<10$" is :
(A) $2+3 \neq 5$ and <10
(B) $2+3=5$ and $8 \nless 10$
(C) $2+3 \neq 5$ or $8 \nless 10$
(D) None of these

***5.** The conditional $(p \wedge q) \Rightarrow p$ is :
(A) A tautology
(B) A fallacy *i.e.*, contradiction
(C) Neither tautology nor fallacy
(D) None of these

6. Which of the following is logically equivalent to
$$\sim(\sim p \Rightarrow q)$$
(A) $p \wedge q$
(B) $p \wedge \sim q$
(C) $\sim p \wedge q$
(D) $\sim p \wedge \sim q$

7. $\sim(p \vee q)$ is equal to :
(A) $\sim p \vee \sim q$
(B) $\sim p \wedge \sim q$
(C) $\sim p \vee q$
(D) $p \vee \sim q$

8. $(\sim(\sim p) \wedge q)$ is equal to :
(A) $\sim p \wedge q$
(B) $p \wedge q$
(C) $p \wedge \sim q$
(D) $\sim p \wedge \sim q$

9. $\sim(\sim p) \wedge q \sim((\sim p) \wedge q)$ is equal to :
(A) $p \vee (\sim q)$
(B) $p \vee q$
(C) $p \wedge (\sim q)$
(D) $\sim p \wedge \sim q$

10. $p \Rightarrow q$ can also be written as :
(A) $p \Rightarrow \sim q$
(B) $\sim p \vee q$
(C) $\sim q \Rightarrow \sim p$
(D) None of these

***11.** $\sim(p \Rightarrow q) \Leftrightarrow \sim p \vee \sim q$ is :
(A) A tautology
(B) A contradiction
(C) Neither a tautology nor a contradiction
(D) Cannot come to any conclusion

***12.** $(p \wedge \sim q) \wedge (\sim p \vee q)$ is :
(A) A contradiction
(B) A tautology
(C) Either (A) or (B)
(D) Neither (A) nor (B)

***13.** The negation of the compound proposition $p \vee (\sim p \vee q)$ is :
(A) $(p \wedge \sim q) \wedge \sim p$
(B) $(p \wedge \sim q) \vee \sim p$
(C) $(p \vee \sim q) \vee \sim p$
(D) None of these

***14.** Which of the following is true :
(A) $p \Rightarrow q \equiv \sim p \Rightarrow \sim q$
(B) $\sim(p \Rightarrow \sim q) \equiv \sim p \wedge q$
(C) $\sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$
(D) $\sim(p \Leftrightarrow q) \equiv [\sim(p \Rightarrow q) \wedge \sim(q \Rightarrow p)]$

15. $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to :
(A) $\sim p$
(B) p
(C) q
(D) $\sim q$

16. The inverse of the proposition $(p \wedge \sim q) \Rightarrow r$ is :
(A) $\sim r \Rightarrow \sim p \vee q$
(B) $\sim p \vee q \Rightarrow \sim r$
(C) $r \Rightarrow p \wedge \sim q$
(D) None of these

17. Let p be the proposition : Mathematics is an interesting and let q be the proposition that Mathematics is difficult, then the symbol $p \wedge q$ means

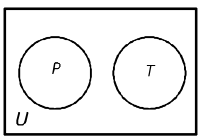
(A) Mathematics is interesting implies that Mathematics is difficult

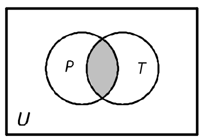
(B) Mathematics is interesting implies and is implied by Mathematics is difficult

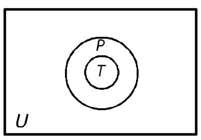
(C) Mathematics is interesting and Mathematics is difficult

(D) Mathematics is interesting or Mathematics is difficult

***18.** Which Venn diagram represents the truth of the statement "No policeman is a thief"

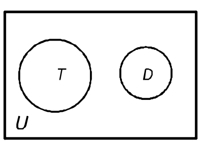
(A) 

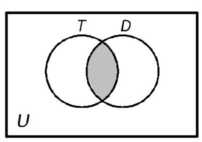
(B) 

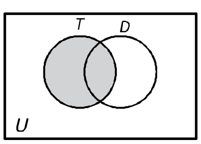
(C) 

(D) None of these

***19.** Which Venn diagram represents the truth of the statement "Some teenagers are not dreamers"

(A) 

(B) 

(C) 

(D) None of these

20. The negative of $q \vee \sim(p \wedge r)$ is :

(A) $\sim q \wedge \sim(p \wedge r)$ (B) $\sim q \wedge (p \wedge r)$

(C) $\sim q \vee (p \wedge r)$ (D) None of these

***21.** The propositions $(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$ is a:

(A) Tautology and contradiction

(B) Neither tautology nor contradiction

(C) Contradiction

(D) Tautology

22. Which of the following is always true :

(A) $(p \Rightarrow p) \equiv \sim q \Rightarrow \sim p$

(B) $\sim(p \vee q) \equiv \vee\ p \vee \sim q$

(C) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$

(D) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

23. The contrapositive of $(p \vee q) \Rightarrow r$ is:

(A) $r \Rightarrow (p \vee q)$ (B) $\sim r \Rightarrow (p \vee q)$

(C) $\sim r \Rightarrow \sim p \wedge \sim q$ (D) $p \Rightarrow (p \vee r)$

***24.** If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively :

(A) T, F, F (B) F, F, F

(C) F, T, T (D) T, T, F

25. The logically equivalent proposition of $p \Leftrightarrow q$ is :

(A) $(p \wedge q) \vee (p \wedge q)$ (B) $(p \Rightarrow q) \wedge (q \Rightarrow p)$

(C) $(p \wedge q) \vee (q \Rightarrow p)$ (D) $(p \wedge q) \Rightarrow (q \vee p)$

***26.** The false statement in the following is :

(A) $p \wedge (\sim q)$ is a contradiction

(B) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction

(C) $\sim(\sim p) \Leftrightarrow p$ is a tautology

(D) $p \vee (\sim p) \Leftrightarrow$ is a tautology

***27.** If $p \Rightarrow (\sim p \vee q)$ is false, the truth values of p and q are respectively :

(A) F, T (B) F, F

(C) T, T (D) T, F

28. Which of the following is not a proposition :

(A) $\sqrt{3}$ is a prime

(B) $\sqrt{2}$ is irrational

(C) Mathematics is interesting

(D) 5 is an even integer

***29.** $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is :

(A) A tautology

(B) A contradiction

(C) Both a tautology and a contradiction

(D) Neither a tautology nor a contradiction

30. $\sim p \wedge q$ is logically equivalent to **:**

(A) $p \to q$ (B) $q \to p$

(C) $\sim (p \to q)$ (D) $\sim (q \to p)$

31. Which of the following is the inverse of the proposition : "If a number is a prime then it is odd."

(A) If a number is not a prime then it is odd

(B) If a number is not a prime then it is odd

(C) If a number is not odd then it is not a prime

(D) If a number is not odd then it is a prime

ANSWERS TO IN-CHAPTER EXERCISE

1.(i) True **(ii)** False **(iii)** False **(iv)** False **(v)** True

2.(i) The square of 4 is not 16 **(ii)** 14 does not divide 27 **(iii)** Chandigarh is not in Gujarat **(iv)** $7 \le 3$

3.(i) False **(ii)** True **(iii)** False **(iv)** True

4.(i) Lucknow is in U.P.and 4 divides 12 **(ii)** Lucknow is in U.P. or 4 divides 12
(iii) Lucknow is not in U.P. and 4 divides 12 **(iv)** Lucknow is not in U.P. or 4 divides 12
(v) Lucknow is in U.P. and 4 does not divide 12 **(vi)** Lucknow is in U.P. or 4 does not divide 12
(vii) Lucknow is not in U.P. and 4 does not divide 12 **(viii)** Lucknow is not in U.P. or 4 does not divide 12

5.(i) A line is straight. A line extends indefinitely in both directions. True
(ii) 0 is less than evey positive integer. 0 is less than every negative integer. False
(iii) All living things have two legs. All living things have two eyes. False

6.(i) True **(ii)** False **(iii)** False **(iv)** False

7.

p	q	r	(i)	(ii)	(iii)	(iv)
T	T	T	T	F	F	F
T	T	F	F	T	T	F
T	F	T	T	F	F	T
F	T	T	T	T	F	T
T	F	F	T	F	T	F
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	F	F	T	F

8.(i) '3 < 4' if and only if '7 > 5' **(ii)** '3 $\ge$ 4' if and only if '7 > 5' **(iii)** '3 < 4' if and only if '$7 \le 5$'
(iv) '$3 \ge 4$' if and only if '$7 \le 5$'

9.(i) False **(ii)** True **(iii)** True **(iv)** True

10.

p	q	(i)	(ii)
T	T	F	T
T	F	T	F
F	T	T	F
T	F	F	T

11.

p	q	r	$(p \vee q) \leftrightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
F	T	T	T
T	F	F	F
F	T	F	F
F	F	T	F
F	F	F	T

17.(i) $[(p \wedge r) \wedge (\sim p \wedge \sim q)] \vee r$ **(ii)** $[(p \wedge q) \vee \sim r] \wedge (p \vee t)$

18. Valid **19.** Valid **20.** Valid

ANSWERS TO OBJECTIVE WORKSHEET

1. D	**2.** A	**3.** B	**4.** C	**5.** A	**6.** D	**7.** B
8. B	**9.** A	**10.** B	**11.** C	**12.** A	**13.** A	**14.** C
15. A	**16.** B	**17.** C	**18.** A	**19.** C	**20.** B	**21.** C
22. C	**23.** C	**24.** A	**25.** B	**26.** B	**27.** D	**28.** C
29. B	**30.** D	**31.** B				